JOURNAL OF THERMOPHYSICS AND HEAT TRANSFER
Vol. 20, No. 3, July–September 2006

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Thermally Developing Flow in Microchannels

Orhan Aydın* and Mete Avcı†
Karadeniz Technical University, 61080 Trabzon, Turkey

I. Introduction

THERE has been an increasing research interest in understanding transport phenomena at microscale because of the rapid growth of techniques applied in microelectromechanical systems and bioengineering. Readers are referred to see recent excellent reviews by Ho and Tai,[1] Palm,[2] Sobhan and Garimella,[3] Obot,[4] Rostami et al.,[5,6] Gad-el-Hak,[7] Guo and Li,[8,9] Morini,[10] and Gravesen et al.[11]

Differing from macroscale case, the hydrodynamic slip and the thermal temperature jump conditions arise at the microscale case as a result of the rarefied gas flow. Viscous dissipation is another effect that should be taken into consideration at microscale. It changes the temperature distributions by playing a role like an energy source as a result of a considerable power generation induced by the shear stress, which, in turn, affects heat-transfer rates.

Gaseous flow in two-dimensional micromachined channels for various Knudsen numbers was studied by Harley et al.[12] Barron et al.[13] extended the Graetz problem to slip flow and developed simplified relationships to describe the effect of slip flow on the convection heat-transfer coefficient. Ameel et al.[14] analytically treated the problem of laminar gas flow in microtubes with a constant heat-flux boundary condition at the wall assuming a slip-flow hydrodynamic condition and a temperature jump thermal condition. Zhu et al.[15] conducted a theoretical analysis of heat-transfer characteristics in a parallel plate channel with microspacing using the boundary condition of the slip velocity and the temperature jump. In a recent study, Aydın and Avcı[16] investigated the effect of the viscous dissipation on the heat transfer for a hydrodynamically developed but thermally developing Poiseuille flow.

The objective of the present study is to theoretically investigate hydrodynamically fully developed but thermally developing rarefied gas flow in a microchannel between two parallel plates. The interactive influences of the Brinkman number and the Knudsen number on convective laminar heat transfer in the thermal entrance region are studied for different thermal boundary conditions at wall.

II. Analysis

We study steady, laminar, hydrodynamically fully developed but thermally developing flow at microscale case including the hydrodynamic velocity slip and the thermal temperature jump conditions at the wall and considering the viscous dissipation in the fluid. The thermal conductivity and the thermal diffusivity of the fluid are considered to be independent of temperature. The axial heat conduction in the fluid and in the wall is assumed to be negligible.

The hydrodynamic velocity slip condition states that[7]

$$u_s = -\frac{2-F}{F}\lambda \frac{\partial u}{\partial y}\bigg|_{y=w} \tag{1}$$

where u_s, λ, and F represent the slip velocity, the molecular mean free path, and the tangential momentum accommodation coefficient, respectively. The thermal temperature jump condition is defined as[7]

$$T_s - T_w = -\frac{2-F_t}{F_t}\frac{2\gamma}{\gamma+1}\frac{\lambda}{Pr}\frac{\partial T}{\partial y}\bigg|_{y=w} \tag{2}$$

where T_s is the temperature of the gas at the wall, T_w the wall temperature, and F_t is the thermal accommodation coefficient. For the rest of the analysis, F and F_t will be shown by F and assumed to be 1.

The fully developed velocity profile taking the slip-flow condition at the wall into consideration is derived from the momentum equation as

$$\frac{u}{u_m} = \frac{3}{2}\left[\frac{1-(y/w)^2+4Kn}{1+6Kn}\right] \tag{3}$$

where u_m is the mean velocity and Kn is the Knudsen number, $Kn = \lambda/2w$.

The conservation of energy including the effect of the viscous dissipation requires

$$u\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

where the second term in the right-hand side is the viscous dissipation term.

Because of axisymmetry at the center, the thermal boundary condition at $y = 0$ can be written as

$$\frac{\partial T}{\partial y}\bigg|_{y=0} = 0 \tag{5}$$

Thermal boundary conditions of constant wall heat flux $H1$ and constant wall temperature T at wall are considered, and they are separately treated in the following sections.

A. Constant Wall Temperature *T*

Introducing the following dimensionless variables

$$U = u/u_m, \qquad \theta = (T_s - T)/(T_s - T_e), \qquad Y = y/W$$
$$Z = (z/w)/RePr \tag{6}$$

where Re is the Reynolds number based on this mean velocity and the distance between plates W, which is equal to $2w$, and Z is termed the reciprocal Graetz number, then Eq. (4) becomes

$$U\frac{\partial\theta}{\partial Z} = \frac{\partial^2\theta}{\partial Y^2} - Br\frac{144Y^2}{(1+6Kn)^2} \tag{7}$$

Received 7 July 2005; revision received 28 September 2005; accepted for publication 19 October 2005.

*Associate Professor, Department of Mechanical Engineering; oaydin@ktu.edu.tr.

†Research Assistant, Department of Mechanical Engineering; mavci@ktu.edu.tr.

where Br is the Brinkman number, which is defined as

$$Br = \mu u_m^2 \big/ k(T_s - T_e) \tag{8}$$

For $z > 0$,

$$T = T_s \quad \text{at} \quad y = w \tag{9}$$

In dimensionless form, the thermal boundary conditions that will be applied in the solution of the energy equations are given as

$$Y = 0: \quad \frac{\partial \theta}{\partial Y} = 0 \qquad Y = 0.5: \quad \theta = 0 \tag{10}$$

The local Nusselt number is obtained from

$$Nu_w = -\frac{\partial \theta}{\partial Y}\bigg|_{y=0.5} \tag{11}$$

which is based on the difference between the wall and the inlet temperatures, that is, on $T_s - T_e$, and not on the difference between the wall and the mean temperatures. The mean temperature, that is, the bulk mean temperature, is given by[17]

$$T_m = \frac{\int \rho u T \,\mathrm{d}A}{\int \rho u \,\mathrm{d}A} \tag{12}$$

Rewriting this equation in terms of the dimensionless variables:

$$\frac{T_s - T_m}{T_s - T_e} = \frac{16}{11}\int_0^{0.5} U\theta \,\mathrm{d}Y \tag{13}$$

$$\frac{T_w - T_m}{T_s - T_e} = \frac{T_s - T_m}{T_s - T_e} - \frac{T_s - T_w}{T_s - T_e} = \frac{16}{11}\int_0^{0.5} U\theta \,\mathrm{d}Y - \frac{2\gamma}{\gamma+1}\frac{Kn}{Pr}\frac{\partial\theta}{\partial Y}\bigg|_{Y=0.5} \tag{14}$$

The Nusselt number based on the difference between the wall and the mean temperature is then given by

$$Nu_{\mathrm{Wm}} = Nu_W \frac{T_s - T_e}{T_w - T_m} = Nu_W \Bigg/ \left(\frac{16}{11}\int_0^{0.5} U\theta \,\mathrm{d}Y - \frac{2\gamma}{\gamma+1}\frac{Kn}{Pr}\frac{\partial\theta}{\partial Y}\bigg|_{Y=0.5}\right) \tag{15}$$

B. Constant Heat-Flux Case $H1$

In this case, the dimensionless temperature is defined as follows:

$$\theta_q = (T - T_e)/(q_w W/k) \tag{16}$$

Using this definition, the energy equation can be expressed in dimensionless form as

$$U\frac{\partial \theta}{\partial Z} = \frac{\partial^2 \theta}{\partial Y^2} + Br_q \frac{144Y^2}{(1+6Kn)^2} \tag{17}$$

where Br_q is the modified Brinkman number, which is given as

$$Br_q = \mu u_m^2 \big/ q_w W \tag{18}$$

The entrance condition at the beginning of the thermally developing region in this case is defined as

$$Z = 0: \quad \theta = 0 \tag{19}$$

and the thermal boundary condition at the wall is

$$k\frac{\partial T}{\partial y}\bigg|_{y=w} = q_w \tag{20}$$

where q_w is positive when its direction is to the fluid (wall heating); otherwise, it is negative (wall cooling). In dimensionless form, it can be written as

$$\frac{\partial \theta_q}{\partial Y}\bigg|_{Y=0.5} = 1 \tag{21}$$

For this case, the Nusselt number is given:

$$\theta_{q,s} = (T_s - T_e)/(q_w W/k) = 1/Nu_w \tag{22}$$

and

$$\theta_{q,m} = \frac{T_m - T_e}{q_w W/k} = \frac{16}{11}\int_0^{0.5} U\theta \,\mathrm{d}Y \tag{23}$$

$$\frac{T_m - T_w}{q_w W/k} = \frac{T_s - T_m}{q_w W/k} - \frac{T_s - T_w}{q_w W/k} = \theta_{q,s} - \theta_{q,m} + \frac{2\gamma}{\gamma+1}\frac{Kn}{Pr} \tag{24}$$

$$Nu_{\mathrm{Wm}} = 1 \Bigg/ \left(\theta_{q,s} - \theta_{q,m} + \frac{2\gamma}{\gamma+1}\frac{Kn}{Pr}\right) \tag{25}$$

For each case, the energy equation has been solved numerically using the finite difference method. The details of the solution procedure can be found in Ref. 17.

III. Results and Discussion

In this study, we investigate the thermally developing flow in a microchannel between two parallel plates. At first, we obtain results at some limiting values of the governing parameters, for which data are available in the existing literature. For the constant heat flux at the wall $H1$, neglecting the viscous dissipation effect ($Br_q = 0$), the local Nusselt-number values are compared with those given by Cotta and Özışık,[18] and a good agreement is obtained (see Table 1).

As stated earlier, the Brinkman number representing the viscous dissipation effect and the Knudsen number representing the rarefaction effects are the two main parameters governing heat and fluid flow in the microchannel. Note that $Kn = 0$ represents the macroscale case, while $Kn > 0$ holds for the microscale case. $Br = 0$ or $Br_q = 0$ represents the case without the effect of the viscous dissipation. Two different thermal boundary conditions have been examined, namely, constant heat flux $H1$ and constant wall temperature T, separately in the following.

For the constant wall temperature condition T, at wall, Fig. 1 depicts the downstream variation of the Nusselt number for different Brinkman numbers at a) $Kn = 0$ and b) $Kn = 0.05$. At $Br = 0$, Nu_{Wm} immediately settles into its usual steady-state value, $Nu_{\mathrm{Wm}} = 4.118$. For the cold wall (i.e., the wall cooling) case, the viscous dissipation increases the temperature difference between the wall and the bulk fluid by increasing the fluid temperature more. As a result, increasing Br in the negative direction increases heat transfer. This situation is an indication of the aiding effect of the viscous dissipation on heat transfer for the wall-cooling case. For the hot wall or the wall-heating case, the Brinkman number will receive

Table 1 Local Nusselt-number values for the $H1$-type ($Br_q = 0.0$, $Kn = 0.0$, $Pr = 0.71$)

Z	Present	Ref. 18
0.000001	149.02	148.78
0.000010	69.176	69.011
0.000100	32.264	32.156
0.001000	15.487	15.427
0.010000	8.8072	8.8031
0.050000	8.2358	8.2355
0.150000	8.2356	8.2353
0.200000	8.2354	8.2353

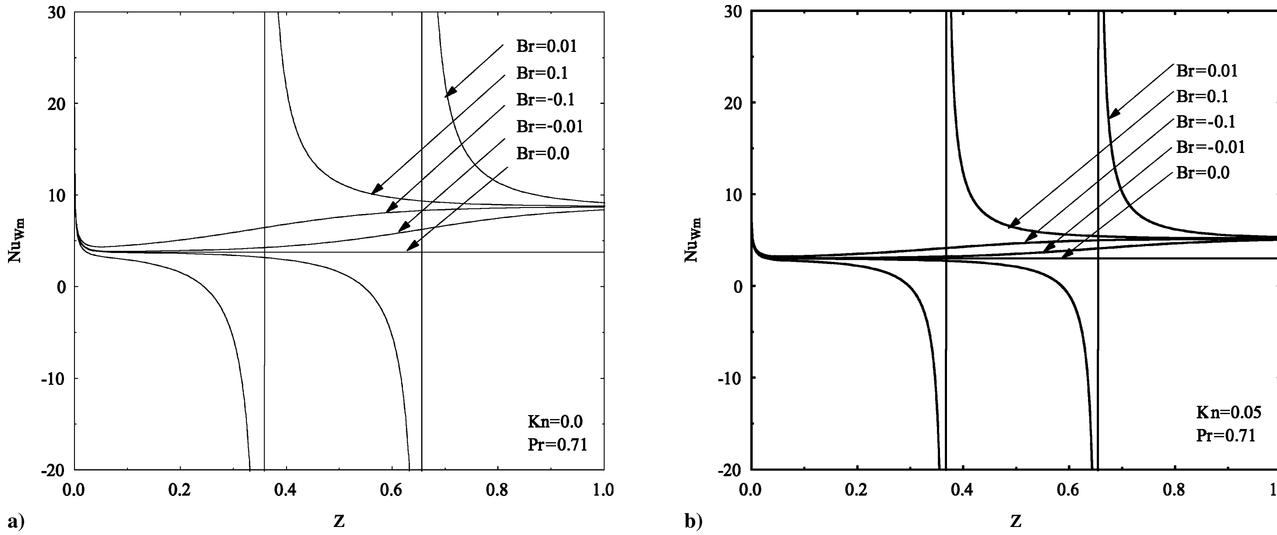

Fig. 1 Variation of Nu_{Wm} with Br for constant wall temperature T type: a) $Kn = 0.0$ and b) $Kn = 0.05$.

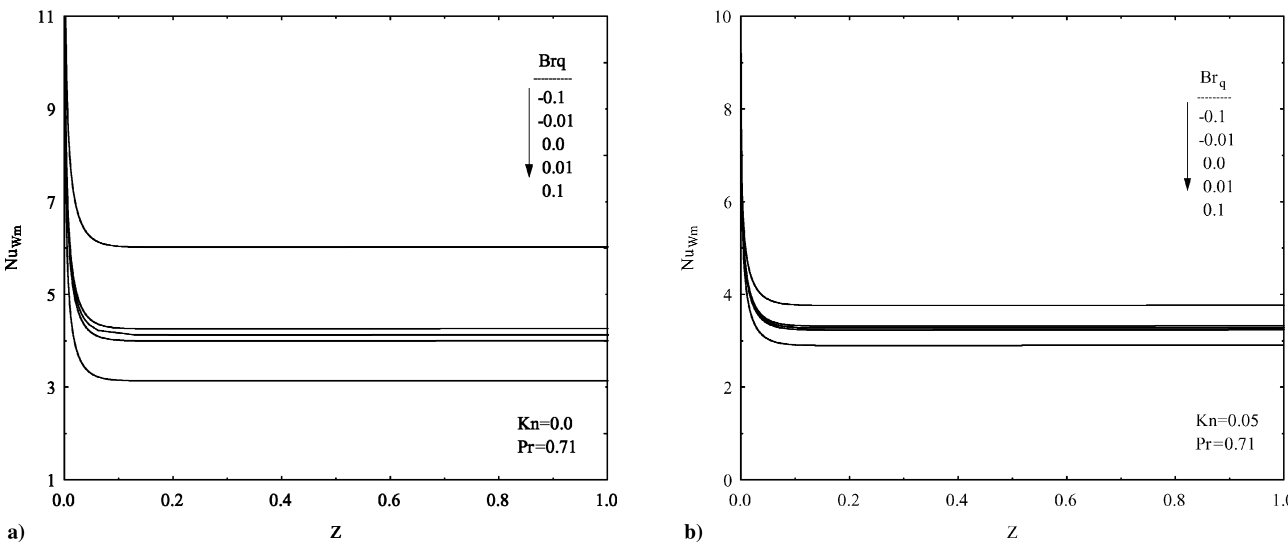

Fig. 2 Variation of Nu_{Wm} with Br_q for constant heat-flux $H1$ type: a) $Kn = 0.0$ and b) $Kn = 0.05$.

positive values, and viscous dissipation will decrease the temperature difference between the wall and the bulk fluid by increasing the bulk fluid temperature. For $Br = 0.01$, Nu_{Wm} decreases up to a singular point, where the internally generated heat caused by viscous dissipation balances the heat supplied by the wall. Beyond the singular point, viscous dissipation heat suppresses the wall heat. At $Br = 0.1$, the singularity is reached at an earlier point in the downstream. Interestingly, in the presence of the viscous dissipation ($Br \neq 0$), the steady-state value of the Nusselt number does not change for any values of the Brinkman number at $Kn = 0$, which is $Nu_{\mathrm{Wm}} = 8.741$. At $Kn = 0.05$, Nu_{Wm} vs Z suggests similar behavior as $Kn = 0$, with lower Nusselt-number values. An increase in Kn decreases the steady-state values of Nu_{Wm}, $Nu_{\mathrm{Wm}} = 3.005$ at $Br = 0$ and $Nu_{\mathrm{Wm}} = 5.174$ at $Br \neq 0$ for $Kn = 0.05$. Inclusion of the temperature jump through the Knudsen number decreases heat transfer. Table 2 illustrates the local Nusselt numbers for different combination of Kn and Br.

For the constant heat-flux condition at the wall $H1$, Fig. 2 shows the downstream variation of the Nusselt number for different values of the modified Brinkman numbers at a) $Kn = 0.0$ and b) $Kn = 0.05$. As expected, increasing dissipation increases the bulk temperature of the fluid caused by internal heating of the fluid. For the wall-heating case, this increase in the fluid temperature decreases the temperature difference between the wall and the bulk fluid, which is followed with a decrease in heat transfer. When wall cooling is applied, because of the internal heating effect of the viscous dissipation on the fluid temperature profile, temperature difference is

Table 2 Fully developed Nusselt-number values at different values of Br for T-type ($Pr = 0.71$)

	Br	
Kn	0.0	$\neq 0$
0.00	3.771	8.741
0.01	3.596	7.693
0.02	3.433	6.861
0.03	3.280	6.191
0.04	3.138	5.638
0.05	3.005	5.174
0.06	2.882	4.780
0.07	2.766	4.441
0.08	2.658	4.146
0.09	2.558	3.887
0.10	2.464	3.658
0.11	2.376	3.455
0.12	2.294	3.272

increased with the increasing Br_q, which, in turn, increases the heat transfer as seen from the figure. Similar to the constant wall temperature case, $Kn = 0.05$ suggests lower heat-transfer values than $Kn = 0$ (Fig. 2b). The sole effect of the Knudsen number can be best seen by neglecting the viscous dissipation effect. Figure 3 illustrates downstream variation of the Nu_{Wm} for different values of Kn in the case without the viscous dissipation effect. Figure 3a is for the constant wall temperature case T, whereas the constant heat-flux case

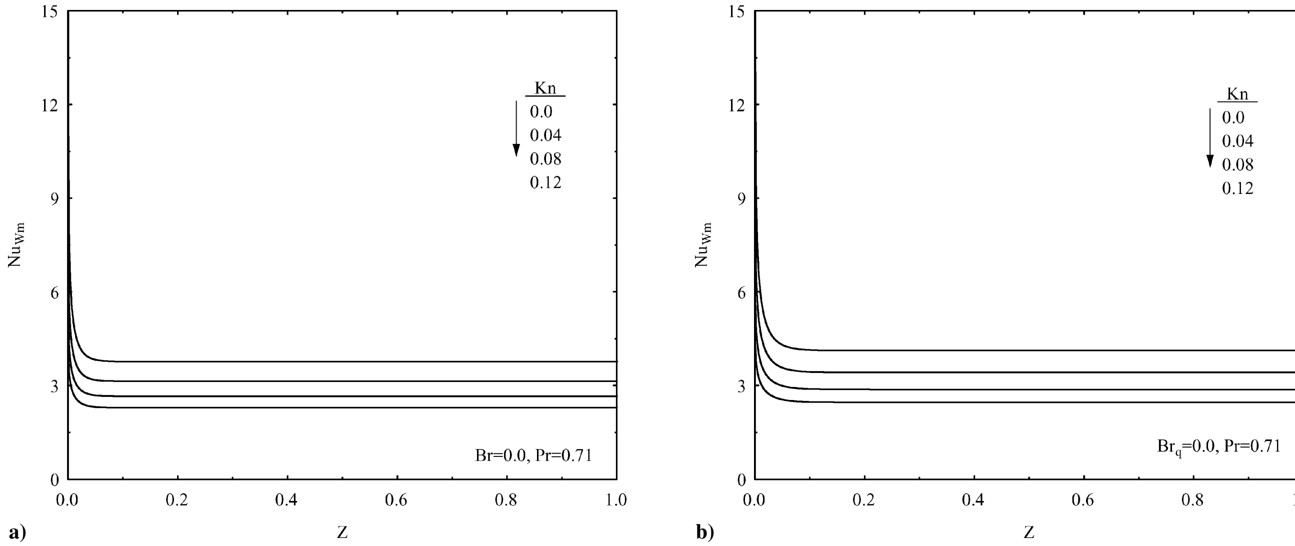

Fig. 3 Variation of Nu_{Wm} with Kn: a) T type and b) $H1$ type.

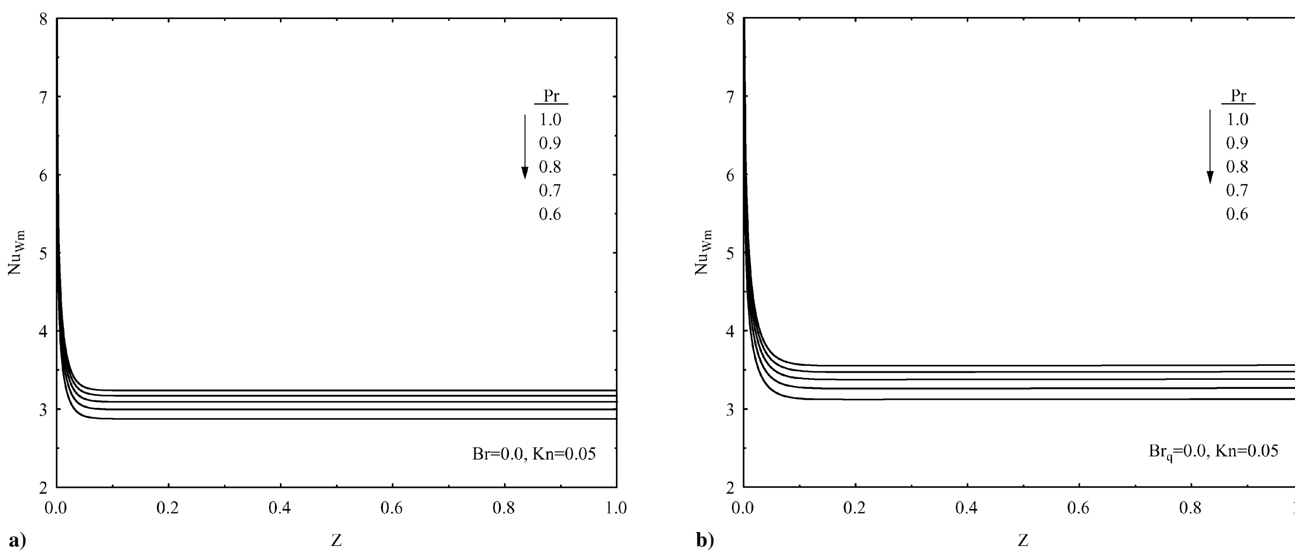

Fig. 4 Variation of Nu_{Wm} with Pr: a) T type and b) $H1$ type.

Table 3 Fully developed Nusselt-number values at different values of Br_q for T-type ($Pr = 0.71$)

	Br_q				
Kn	−0.1	−0.01	0.0	0.01	0.1
0.00	6.009	4.266	4.118	4.008	3.150
0.01	5.305	4.046	3.930	3.844	3.137
0.02	4.780	3.842	3.750	3.682	3.099
0.03	4.371	3.656	3.580	3.528	3.047
0.04	4.036	3.479	3.421	3.376	2.977
0.05	3.760	3.320	3.271	3.236	2.904
0.06	3.524	3.171	3.131	3.103	2.826
0.07	3.320	3.034	3.000	2.977	2.745
0.08	3.140	2.905	2.878	2.858	2.662
0.09	2.981	2.786	2.764	2.747	2.581
0.10	2.838	2.676	2.657	2.643	2.502
0.11	2.711	2.574	2.557	2.546	2.425
0.12	2.595	2.479	2.464	2.455	2.351

is shown in Fig. 3b. Table 3 illustrates the local Nusselt numbers for different combination of Kn and Br_q.

Figure 4 shows the effect of the Prandtl number on the downstream variation of the Nusselt number for the constant wall temperature $T1$ and constant heat-flux $H1$ cases, respectively. An increase in Pr results in higher transfer values. This behavior can be explained by the fact that a fluid with a larger Prandtl number possesses a larger heat capacity and enhances the heat transfer.

IV. Conclusions

We have theoretically studied thermally developing forced convection flow in a microchannel between two parallel plates. The hydrodynamic slip and the thermal temperature jump conditions at the wall and the viscous dissipation in the fluid have been included in the analysis. Two types of wall thermal boundary condition have been considered, namely, constant heat flux ($H1$ type) and constant wall temperature (T type). For the constant wall temperature, the downstream variation of the Nusselt number presented some singularities at any value of the Knudsen number. These singularities have been disclosed to originate from the thermal energy balance between the wall heat and the viscous dissipation heat during the thermal transport. With the increasing Brinkman number, although the singularity point changes, interestingly, the same asymptotic value has been reached. The Knudsen number has been shown to decrease the Nusselt number. For the case without the viscous dissipation, increasing the Knudsen number from 0 to 0.12 dropped Nusselt number from 4.118 to 2.464 (a decrease of 40%) for the $H1$ case and from 3.771 to 2.294 (a decrease of 39%) for the constant wall-temperature case.

References

[1]Ho, C. M., and Tai, C. Y., “Micro-Electro-Mechanical Systems (MEMS) and Fluid Flows,” *Annual Review of Fluid Mechanics*, Vol. 30, 1998, pp. 579–612.

[2]Palm, B., "Heat Transfer in Microchannels," *Microscale Thermophysical Engineering*, Vol. 5, No. 3, 2001, pp. 155–175.

[3]Sobhan, C. B., and Garimella, S. V., "A Comparative Analysis of Studies on Heat Transfer and Fluid Flow in Microchannels," *Microscale Thermophysical Engineering*, Vol. 5, No. 4, 2001, pp. 293–311.

[4]Obot, N. T., "Toward a Better Understanding of Friction and Heat/Mass Transfer in Microchannels-A literature Review," *Microscale Thermophysical Engineering*, Vol. 6, No. 3, 2002, pp. 155–173.

[5]Rostami, A. A., Saniei, N., and Mujumdar, A. S., "Liquid Flow and Heat Transfer in Microchannels: A Review," *Heat Technology*, Vol. 18, No. 2, 2000, pp. 59–68.

[6]Rostami, A. A., Mujumdar, A. S., and Saniei, N., "Flow and Heat Transfer for Gas Flowing in Microchannels: A Review," *Heat Mass Transfer*, Vol. 38, No. 4-5, 2002, pp. 359–367.

[7]Gad-el-Hak, M., "Flow Physics in MEMS," *Mecanique et Industries*, Vol. 2, 2001, pp. 313–341.

[8]Guo, Z. Y., and Li, Z. X., "Size Effect on Microscale Single-Phase Flow and Heat Transfer," *International Journal of Heat and Mass Transfer*, Vol. 46, No. 1, 2003, pp. 149–159.

[9]Guo, Z. Y., and Li, Z. X., "Size Effect on Single-Phase Channel Flow and Heat Transfer at Microscale," *International Journal of Heat and Fluid Flow*, Vol. 24, No. 3, 2003, pp. 284–298.

[10]Morini, G. L., "Single-Phase Convective Heat Transfer in Microchannels: A Review of Experimental Results," *International Journal of Thermal Sciences*, Vol. 43, No. 7, 2004, pp. 631–651.

[11]Gravesen, P., Branebjerg, J., and Jensen, O. S., "Microfluidics—A Review," *Journal of Micromechanics and Microengineering*, Vol. 3, No. 4, 1993, pp. 168–182.

[12]Harley, J. C., Huang, Y., Bau, H. H., and Zemel, J. N., "Gas Flow in Micro-Channels," *Journal of Fluid Mechanics*, Vol. 284, Feb. 1995, pp. 257–274.

[13]Barron, R. F., Wang, X., Ameel, T. A., and Warrington, R. O., "The Graetz Problem Extended to Slip-Flow," *International Journal of Heat and Mass Transfer*, Vol. 40, No. 8, 1997, pp. 1817–1823.

[14]Ameel, T. A., Wang, X. M., Barron, R. F., and Warrington, R. O., "Laminar Forced Convection in a Circular Tube with Constant Heat Flux and Slip Flow," *Microscale Thermal Engineering*, Vol. 1, No. 4, 1997, pp. 303–320.

[15]Zhu, X., Xin, M. D., and Liano, Q., "Analysis of Heat Transfer Between Two Unsymmetrically Heated Parallel Plates with Microspacing in the Slip Flow Regime," *Microscale Thermophysical Engineering*, Vol. 6, No. 4, 2002, pp. 287–301.

[16]Aydın, O., and Avcı, M., "Viscous dissipation Effects on the Heat Transfer in a Poiseuille Flow," *Applied Energy* (to be published).

[17]Oosthuizen, P. H., and Naylor, D., *Introduction to Convective Heat Transfer Analysis*, McGraw–Hill, New York, 1999, pp. 158–167.

[18]Cotta, R. M., and Özısık, M. N., "Laminar Forced Convection to Non-Newtonian Fluids in Ducts with Prescribed Wall Heat Flux," *International Communications in Heat and Mass Transfer*, Vol. 13, No. 3, 1986, pp. 325–334.